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INVESTIGATING THE STABILITY OF THE STATIONARY REGIME OF THE AUTONOMOUS SYNCHRONOUS GENERATOR WITH THE ACTIVE-INDUCTIVE LOAD

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The system of the autonomous synchronous generator with an active-inductive load is considered. The mathematical model of the transient process in the coordinate system d, q is presented, as well as the stationary regime of the autonomous synchronous generator is introduced. The conditions for the existence and the parameters of the stationary regime are obtained. A mathematical model of the transient process of the autonomous synchronous generator in a canonical form is presented. For the mathematical model in the canonical form, the Jacobi matrix is obtained and its stability in the stationary points is studied. The stability was assessed using the first theorem of Lyapunov. For investigating the stability, a numerical example is considered whose results are obtained by using the software package Matlab.

Keywords: autonomous synchronous generator, transient process, mathematical model, stationary regime, condition for the existence.

Introduction. The mathematical model of a synchronous generator (SG) transient process is described by a system of ordinary nonlinear differential equations [1]. When the SG is operating isolated from the electrical network, i.e. by its own load, it is called an autonomous synchronous generator (ASG). The static stability investigation of the stabilized regime of ASG leads to the adequate solution of nonlinear differential equations stability control system. The stability criteria for solving the system of differential equations are defined by famous theorems of Lyapunov. For solving practical problems, Lyapunov's first theorem on stability is applied.

Let us consider the most famous works [1-3] among the ones devoted to the ASG's stationary regime of the transient process and transformation of mathematical models for investigating their stability and peculiarities.

In work [1], the investigation of the self-excitation ASG's with an active - inductive load is introduced (parametric resonance) and it is not related to the issues considered in our work.

In work [2], the stability of ASG's stationary regime is not investigated.

In the fundamental work [3] devoted to the research of static and dynamic stability parameters of the electrical system and electrical machines, as well as the impacts of various factors, the case of autonomous synchronous machines is not considered.

Obtaining the ASG mathematical model. *The goal of the work is to* investigate the stability of the ASG stationary regime.

Let us consider an ASG with implicit poles, one excitation winding and without damping windings schematically shown in Fig.1 and obtain the mathematical model of the transient process.

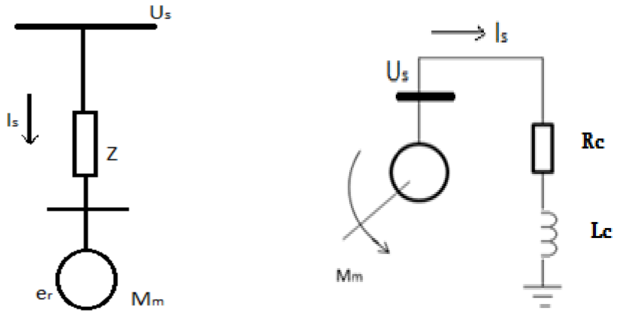

Fig. 1. a) SG's scheme connected to the network, b)Scheme of the ASG: Rc,Lc load parameters

For obtaining the mathematical model of the ASG's transient process, let us consider the mathematical model of the SG connected to the network which is schematically shown in Fig.1.

The voltages, in the rotor and stator windings are determined by the following relationships [1]:

$$\begin{cases} U_S = R_S i_S + \frac{d\psi_S}{dt}, \\ E_r = r_r i_r + \frac{d\psi_r}{dt}, \end{cases} \quad S = a,b,c, \tag{1}$$

where U_S is the stator voltage; $R_s(r_r)$ – the resistances of the stator (rotor) windings; $i_S(i_r)$ – the currents of the stator (rotor) windings; ψ_S (ψ_r) – the flux linkages of the stator (rotor) windings; Er – the constant voltage applied to the rotor winding.

The winding flux linkages through the currents will be expressed by the following formulae [1]:

$$\begin{cases} \psi_A = (L-M)i_A + L_m i_r cos\alpha, \\ \psi_B = (L-M)i_B + L_m i_r cos(\alpha - 120), \\ \psi_C = (L-M)i_C + L_m i_r cos(\alpha + 120), \\ \psi_r = l i_r + L_m[i_A cos\alpha + i_B cos(\alpha - 120) + i_C cos(\alpha + 120)], \end{cases} \tag{2}$$

where $L(l)$ is the inductance of stator (rotor) windings; M – the mutual inductance between the phase windings of the stator; L_m – the mutual inductance between the phases of the stator and the rotor windings at their parallel arrangement; $\alpha = \alpha_0 + \omega t$ – the angle between the A phase of the stator and rotor winding; ω – the angular velocity of the rotor. In the electrical power system, the synchronous machine angular velocities of rotors may be different from the voltage angular frequency in asynchronous regimes. In the ASGs, the angular velocity of the rotor and the voltage angular frequency of the stator are equal in any regime.

The system of equations (2) can be presented in the form of a matrix:

$$\begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_r \end{bmatrix} = \begin{bmatrix} L-M & 0 & 0 & L_m\cos\alpha \\ 0 & L-M & 0 & L_m\cos(\alpha-120) \\ 0 & 0 & L-M & L_m\cos(\alpha+120) \\ L_m\cos\alpha & L_m\cos(\alpha-120) & L_m\cos(\alpha+120) & l \end{bmatrix} \cdot \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_r \end{bmatrix}, \quad (3)$$

or

$$\Psi = A \cdot i,$$

where the determinant of matrix A is as follows:

$$\det A = (L-M)^2[(L-M)l - L_m^2\cos^2(\alpha+120) - L_m^2\cos^2(\alpha-120) - L_m^2\cos^2\alpha] =$$
$$= (L-M)^2[(L-M)l - L_m^2(\cos^2(\alpha+120) + \cos^2(\alpha-120) + \cos^2\alpha)] =$$
$$= (L-M)^2\left[(L-M)l - \frac{3}{2}L_m^2\right] \neq 0.$$

Solving the (3) matrix system for the vector of current we will have.

$$i = A^{-1} \cdot \Psi. \quad (4)$$

The resulting equations show that having the currents of the stator and rotor, we can find the corresponding flux linkages and vice versa.

In (1), placing the currents expressions instead of them, in accordance with (4), and adding the equation of Dalamber to the system:

$$J\frac{d\omega}{dt} = M_{el} - M_m \quad (5)$$

where M_{el} is the electromagnetic moment: $M_{el} = -i_r L_m[i_A \sin\alpha + i_B \sin(\alpha - -120) + i_C \sin(\alpha+120)]$, M_m - the outer mechanical moment applied to the rotor, J - the rotor moment of inertia,

we will have the full mathematical model of SG's transient process in the natural form which is a I class system of nonlinear ordinary differential equations with variable coefficients:

$$\begin{cases} \frac{d}{dt}\Psi = -R \cdot A^{-1} \cdot \Psi + E, \\ J\frac{d\omega}{dt} = M_{el} - M_{m,} \end{cases} \quad (6)$$

where $E^t = [U_a, U_b, U_c, U_r]$, $\Psi^t = [\psi_a, \psi_b, \psi_c, \psi_r]$, R is the diagonal matrix of resistances.

The transformation of the natural model into the d, q coordinate system allows to obtain the mathematical model of the transient process of an electrical machine with constant coefficients. The system of mathematical model equations of the transient process of SG connected to the network presented in the d,q coordinate system, has the following form [4]:

$$\begin{cases} R_s i_d + L\frac{di_d}{dt} + M_d\frac{di_r}{dt} + \omega L i_q = U_d, \\ -\omega L i_d + R_s i_q + L\frac{di_q}{dt} - \omega L_m i_r = -U_q, \\ \frac{3}{2}\frac{L_m}{l}\frac{di_d}{dt} + \frac{r_r i_r}{l} + \frac{di_r}{dt} - \frac{E_r}{l} = 0, \\ J\omega\frac{ds}{dt} + \frac{3}{2}L_m i_r i_q - M_m = 0, \end{cases} \tag{7}$$

where $p = \frac{d}{dt}$ is the differentiation operator; U_d, U_q – the stator voltage projection on the d,q axes, i_d, i_q the current projection on the d,q axes.

In case of ASG (Fig. 1), the U_d and U_q values are expressed as follows [5]:

$$\begin{cases} U_d = -R_c i_d - L_c\frac{di_d}{dt} - \omega L_c i_q, \\ U_q = -R_c i_q + L_c\frac{di_q}{dt} - \omega L_c i_d. \end{cases}$$

By including the ASG load shown in Fig.1 in the stator parameter, let us introduce (7) as follows (8):

$$\begin{cases} R_s i_d + L\frac{di_d}{dt} + M_d\frac{di_r}{dt} + \omega L i_q = 0, \\ -\omega L i_d + R_s i_q + L\frac{di_q}{dt} - \omega L_m i_r = 0, \\ \frac{3}{2}\frac{L_m}{l}\frac{di_d}{dt} + \frac{r_r i_r}{l} + \frac{di_r}{dt} - \frac{E_r}{l} = 0, \\ J\omega\frac{ds}{dt} + \frac{3}{2}L_m i_r i_q - M_m = 0, \end{cases} \tag{8}$$

where the load parameters are also included in the Rs and L.

By doing appropriate modifications, we will obtain the mathematical model of ASG's transient process in the nonlinear equation system from (8):

$$\begin{cases} P(i_d + L_m i_r) = \frac{1}{L}\left(-R_s i_d - \omega L i_q\right), \\ P i_q = \frac{1}{L}\left(-R_s i_q + \omega L i_d + \omega L_m i_r\right), \\ P(i_r + \frac{3}{2}\frac{L_m}{l} i_d) = \frac{1}{L}(E_r - r_r i_r), \\ JP\omega = -\frac{3}{2}L_m i_r i_q + M_m. \end{cases} \tag{9}$$

A mathematical model of the ASG's stationary regime. To obtain an ASG's stationary regime, in (9), we admit $p = 0$. The system of equations of stationary motion will have the form:

$$\begin{cases} R_s i_d + \omega L i_q = 0, \\ -R_s i_q + \omega L i_d + \omega L_m i_r = 0, \\ E_r - r_r i_r = 0, \\ -\frac{3}{2} L_m i_r i_q + M_m = 0. \end{cases} \tag{10}$$

By solving the system of equations (10), we obtain the stationary point $(i_{d0}, i_{q0}, i_{r0}, \omega_0)$. From equation 3 of the equations system (10) we obtain i_{r0}, by using which in equation 4, we get i_{q0}. By solving the first and second equations together, we express one variable by the other, and get a quadratic equation. Expressing i_d by ω, we get the quadratic equation (11), otherwise quadratic equation (12) is obtained:

$$\omega^2 \frac{L^2 i_q}{R_s} - \omega L_m i_r + R_s i_q = 0, \tag{11}$$

$$i_d^2 + i_d \frac{L_m i_r}{L} + i_q^2 = 0. \tag{12}$$

The non - negative discriminant of the quadratic equation is a condition for the existence of the stationary point:

$$\left(L_m \frac{E_r}{r_r}\right)^2 - \frac{16}{9}\left(\frac{M_m r_r}{L_m E_r} L\right)^2 \geq 0,$$

or

$$\left(\frac{{E_r}^2}{M_m}\right)^2 \geq \frac{16}{9}\left(\frac{{r_r}^2}{{L^2}_m} L\right)^2. \tag{13}$$

Investigating the stability of the ASG's stationary point. To assess the stability of the ASG's stationary point, let us obtain the Jacobi matrix [6], let us introduce the system of equations (9) in the following form:

$$\begin{bmatrix} 1 & 0 & L_m & 0 \\ 0 & 1 & 0 & 0 \\ \frac{3}{2}\frac{L_m}{l} & 0 & 1 & 0 \\ 0 & 0 & 0 & J \end{bmatrix} x \begin{bmatrix} Pi_d \\ Pi_q \\ Pi_r \\ P\omega \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}, \tag{14}$$

where

$$\begin{cases} f_1 = \frac{1}{L}\left(-R_s i_d - \omega L i_q\right), \\ f_2 = \frac{1}{L}\left(-R_s i_q + \omega L i_d + \omega L_m i_r\right), \\ f_3 = \frac{1}{L}(E_r - r_r i_r), \\ f_4 = -\frac{3}{2} L_m i_r i_q + M_m. \end{cases}$$

Let us introduce the equation system (9) in a vector form:

$$\frac{dz}{dt} = A^{-1} f(z), \qquad (15)$$

where

$$z^t = (i_d, i_q, i_r, \omega),\ A = \begin{bmatrix} 1 & 0 & L_m & 0 \\ 0 & 1 & 0 & 0 \\ \frac{3}{2}\frac{L_m}{l} & 0 & 1 & 0 \\ 0 & 0 & 0 & J \end{bmatrix}.$$

The Jacobi matrix, for the right part of the equation system (15) the vector function will have the following form:

$$Я = \frac{2l}{2Jl - 3L_m^2 J} \cdot \begin{bmatrix} J & 0 & -L_m J & 0 \\ 0 & J - \frac{3L_m^2 J}{2l} & 0 & 0 \\ \frac{3L_m J}{2l} & 0 & J & 0 \\ 0 & 0 & 0 & 1 - \frac{3L_m^2}{2l} \end{bmatrix} \cdot \begin{bmatrix} -\frac{R_s}{L} & -\omega & 0 & -i_q \\ \omega & -\frac{R_s}{L} & \frac{\omega L_m}{L} & i_d + \frac{L_m i_r}{L} \\ 0 & 0 & -\frac{r_r}{L} & 0 \\ 0 & -\frac{3}{2} L_m i_r & -\frac{3}{2} L_m i_q & 0 \end{bmatrix}. \qquad (16)$$

According to Lyapunov's first theorem, the ASG's stationary point is stable if the eigenvalues of the Jacobi matrix have negative real parts. By Raus Hurwitz criterion, it is possible to determine the stability of the Jacobi matrix without determining the eigenvalues [6].

Below is a numerical example, the calculation is performed in the Matlab software system. An ASG with the following parameter is considered.

R_s=10.4827 *Ohm*, r_r=0.8 *Ohm*, Er=0.15 *kV*, L=0.0378 *H*, Lm=0.2085 *H*, Mm=0.03 *MNm.*

In the example, the observed ASG's data have been chosen so that in the stationary regime the voltage frequency of the stator is 314 *rad/sec*. The load parameter is included in the Rs, L.

In accordance with these parameters the 2 stationary points obtained are:

1. Z_1^t= (i_q, i_r ,i_d ,ω)=(0.5115; 0.1875 ; -0,5809; 313.9).
2. Z_2^t= (i_q, i_r ,i_d , ω)=(0.5115; 0.1875 ;-0,4520; 244.2) .

In case of the first stationary point, the Jacobi matrix obtained is:

$$1.0e+003\times\begin{bmatrix}-0.2813 & -0.3193 & -0.0045 & -0.0005\\ 0.3139 & -0.2765 & 1.7261 & 1.7261\\ 0.0230 & 0.0261 & 0.0215 & 0.0000\\ 0.0000 & -0.1173 & -0.3200 & 0.0000\end{bmatrix}.$$

The eigenvalues of the Jacobi matrix are:

-2.8185 e+004, -2.8185 e – 002, -0.2740 , -0.0002.

In case of the second stationary point, the Jacobi matrix obtained is:

$$1.0e+003\times\begin{bmatrix}-0.2813 & -0.2485 & -0.0045 & -0.0005\\ 0.2442 & -0.2765 & 1.3431 & 0.0006\\ 0.0230 & 0.0203 & 0.0215 & 0.0000\\ 0.0000 & -0.1173 & -0.3200 & 0.0000\end{bmatrix}.$$

The eigenvalues of the Jacobi matrix are:

-2.8090 e+004, -2.8090 e –004, 0.2557 , -0.0003 .

The system is stable in case of the first stationary point as the eigenvalues of the Jacobi matrix have negative real parts, while in case of the second stationary point the system is unstable.

Conclusion

1. Including the parameters of the load in the parameters of the stator, we have obtained the mathematical model of ASG's transient process in a d,q form, which is a nonlinear autonomous system of differential equations.
2. For the AGS's stationary regime, a mathematical model and the condition for the existence of the stationary regime are obtained (13).
3. Using the first theorem of Lyapunov, the stability of the ASG's stationary regime is investigated by considering a numerical example.

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ԱԿՏԻՎ-ԻՆԴՈՒԿՏԻՎ ԲԵՌՈՎ ԻՆՔՆԱՎԱՐ ՍԻՆՔՐՈՆ ԳԵՆԵՐԱՏՈՐԻ ՍՏԱՑԻՈՆԱՐ ՌԵԺԻՄԻ ԿԱՅՈՒՆՈՒԹՅԱՆ ՀԵՏԱԶՈՏՈՒՄԸ

Վ.Ս. Սաֆարյան, Մ.Մ. Ղալեչյան

Դիտարկվել է ակտիվ-ինդուկտիվ բեռով ինքնավար սինքրոն գեներատորի աշխատանքը: Հետազոտվել են անցումային գործընթացի մաթեմատիկական մոդելը՝ ներկայացված d,q տեսքով, ինչպես նաև՝ ինքնավար սինքրոն գեներատորի ստացիոնար ռեժիմը: Ստացվել են ստացիոնար ռեժիմի գոյության պայմանը և պարամետրերը: Ներկայացված է ինքնավար սինքրոն գեներատորի անցումային գործընթացների մաթեմատիկական մոդելը՝ կանոնական տեսքով: Ստացվել է կանոնական տեսքի մաթեմատիկական մոդելի Յակոբիի մատրիցը, և հետազոտվել է նրա կայունությունը ստացիոնար կետերում: Կայունությունը գնահատվել է՝ օգտվելով Լյապունովի առաջին թեորեմից: Կայունության հետազոտման համար դիտարկվել է թվային օրինակ, որի արդյունքները ստացվել են Matlab ծրագրային փաթեթի միջոցով:

Առանցքային բառեր. ինքնավար սինքրոն գեներատոր, անցումային գործընթաց, մաթեմատիկական մոդել, ստացիոնար ռեժիմ, գոյության պայման:

ИССЛЕДОВАНИЕ УСТОЙЧИВОСТИ СТАЦИОНАРНОГО РЕЖИМА АВТОНОМНОГО СИНХРОННОГО ГЕНЕРАТОРА С АКТИВНО-ИНДУКТИВНОЙ НАГРУЗКОЙ

В.С. Сафарян, М.М. Галечян

Рассмотрена система автономно работающего синхронного генератора совместно с активно-индуктивной нагрузкой. Исследованы математическая модель переходного процесса, представленная в системе d и q, а также стационарный режим генератора. Получены условия существования и параметры стационарного режима. Рассмотрена математическая модель переходного процесса автономного синхронного генератора в канонической форме. Получена матрица Якоби для математической модели канонической формы и исследована ее устойчивость в стационарных точках. Устойчивость оценивалась с использованием первой теоремы Ляпунова. Для исследования устойчивости рассмотрен численный пример, результаты которого получены с помощью программного пакета Matlab.

Ключевые слова: автономный синхронный генератор, переходный процесс, математическая модель, стационарный режим, условие существования.