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LEAST SQUARE APPROXIMATIONS ON FINITE SETS OF LINES AS APPLIED TO THE SYNTHESIS OF ADJUSTABLE ROBOTIC MECHANISMS

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The problem of determining the special lines of a moving body which in *m* alternating sets of its given positions deviate least, in a least-square sense, from the coaxial line congruences generated by the moving axes of CC dyads with the common fixed axis of rotation is considered. The sought-for approximation is one which minimizes two sums of the squared angular and linear deviations of such lines from the approximating line congruences associated with the CC dyads to be synthesized. Two theorems describing geometrically the necessary conditions for the best least-square approximation of the given *m* line-position sets by coaxial circular cones and line congruences are formulated. The loci of the special lines under consideration are studied, and a method for their determination is proposed. The theory and method presented in the paper can be readily applied to the synthesis of adjustable parallel robotic mechanisms based on C$\underline{C}$C chains (modules) with the lockable middle joints which serve for adjusting the angle and distance between the moving and fixed axes of rotation to realize multiple kinematic tasks. The synthesis of C$\underline{C}$C chain is decomposed into two simpler subproblems: a) synthesis of its spherical indicatrix R$\underline{R}$R with the lockable middle joint to determine directions of the moving and fixed axes of rotation and *m* values of their adjustable twist angle, b) synthesis of C$\underline{C}$C chain with the known directions of the rotation axes to determine their locations in the corresponding coordinate systems and *m* values of the adjustable distance between them.

***Keywords*:** coaxial line congruences, least-square approximation, angular deviation, CC dyad, robotic mechanism.

Introduction. The theory of the kinematic synthesis for Cylindric-Cylindric (CC) dyads to exactly match 3, 4, 5 design positions of the output link has been developed in [1]. In [2], we have considered the same problem for an arbitrary number of given positions, seeking to determine the special CC dyads which approximate, in a least square sense, all given positions. In [3], a method has been proposed to synthesize CC dyads approximating best the finite sets of ordered lines, in a Chebishev sense.

The present paper is an extension of [2] where a single set of design positions has been considered only. Here we discuss the general case of multiple sets of the given positions to be approximated. The problem we set ourselves in this paper is to determine the special lines in moving body *e* which in $m>1$ given sets of its positions best approximate coaxial line congruences generated by the moving axis q of a C<u>C</u>C chain with the lockable middle cylindric joint adjusting the twist angle γ and distance d between the moving and fixed axes q and Q (Fig. 1). This joint is locked in all operation modes of the mechanism by a physical stopper attached to the joint itself. The problem under consideration is directly connected with the synthesis of multimode parallel robotic mechanisms built on the basis of C<u>C</u>C modules with the lockable middle joints shown in Fig.1. Such mechanisms can be used to design reconfigurable manufacturing systems with a short changeover time since they need fewer actuators to realize multiple kinematic tasks and there is no need to disassemble the mechanism in the process of its reconfiguration for a new task [4].

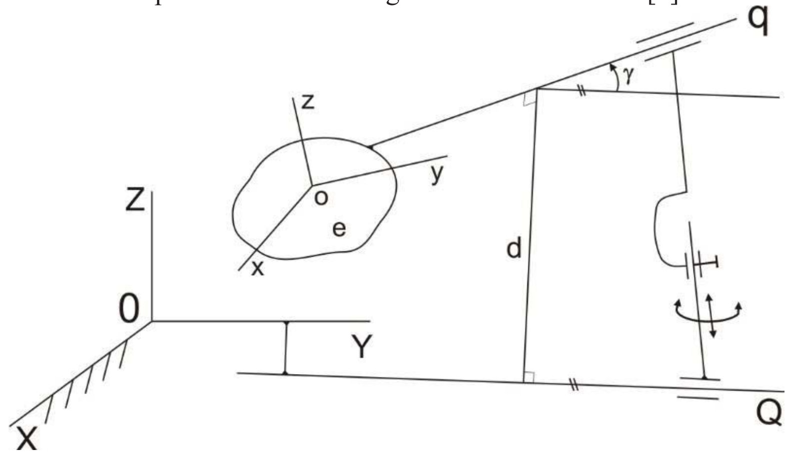

Fig.1. C<u>C</u>C chain with the lockable middle joint

For each pair of fixed values γ_j and d_j (j=1, 2,…, m) of γ and d, the three link C<u>C</u>C chain with its locked middle joint turns into a CC dyad the axis q of which generates a line congruence k_j called in [3] "CC congruence". In what follows we will use this term. For m values of j, we have a set of coaxial line congruences $K = \{k_j\}_{j=1}^{m}$. In other words, we seek to determine a set of CC dyads with the common axis Q which are different from each other by their adjustable parameters γ and d.

The paper is organized as follows. First we consider a simpler problem of approximating the given position sets of a line q in e by coaxial CC congruences which, as will be shown below, can be solved in a closed form. Then, based on the solution of this problem, the special lines of e are determined which are closest, in a least square sense, to the associated sets of approximating coaxial line congruences in the given m positions- sets of e.

Least square approximations of the given line-position sets by coaxial CC congruences. Body e is given in the m sets of finitely separated positions e_{ji} ($j = 1,2,\dots,m;\ i = 1,2,\dots,N_j$) relative to a coinciding fixed body E with N_j positions each. Coordinate systems *oxyz* and *OXYZ* are rigidly attached to e and E respectively (Fig. 2). Here we study the following problem: given a line q in e determine a set K^*

of m CC congruences $K = \{k_j\}_{j=1}^{m}$ with the common central axis Q* which is as close as possible to all the m sets of positions $\{q_{ji}\}_{i=1}^{N_j}$ $(j = 1,2,\dots,m)$ of q, in a sense to be specified.

This problem would have been separable into m problems of approximating a single line-position set by a CC line congruence considered in [2]. However, a common central axis Q for all line congruences K_j is required which makes these problems interconnected. To solve the problem we should define first the deviation and the norm of the sought – for approximation.

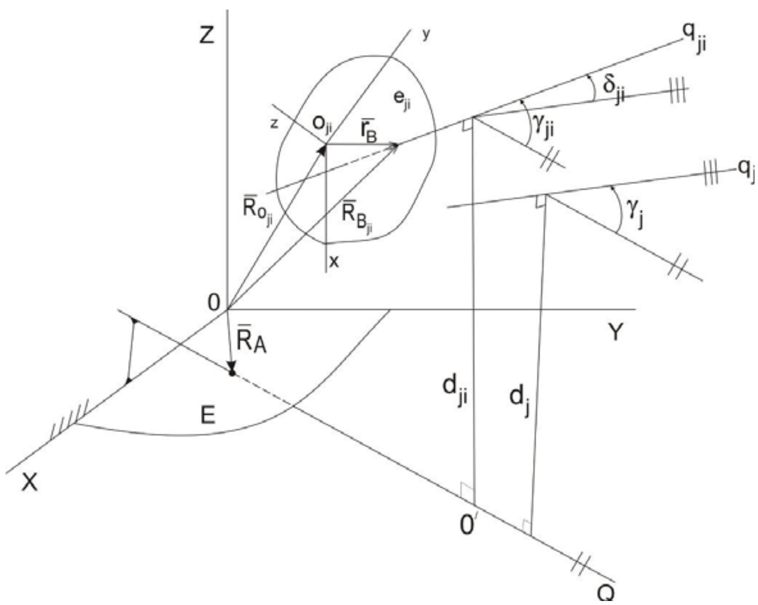

Fig.2. Angular and distance deviations between moving line q and CC congruence k_j

To measure the distance of the given line set to coaxial CC congruences we, will use the so called "dual modulus of deviation" defined in [3]. Since the relative position of two lines depends upon their relative angle and distance, we have an angular modulus and a linear modulus. Thus, the dual modulus will be composed of two components: the angular modulus and the distance modulus.

For the angular deviation δ_{ji} of the line q_{ji} associated with the position e_{ji} of e and j-th CC congruence K_j we have

$$\delta_{ji} = \gamma_{ji} - \gamma_j. \tag{1}$$

For the distance deviation Δ_{ji} we have

$$\Delta_{ji} = d_{ji} - d_j. \tag{2}$$

Here γ_j and d_j are the twist angle and distance of the j-th CC congruence K_j while γ_{ji} and d_{ji} are those of a moving line q in its i-th position of the j-th given position set of e relative to the central axis Q of K_j.

We will define lines q and Qin systemsoxyz and OXYZ by their canonical equations

$$\frac{x-x_B}{q_x} = \frac{y-y_B}{q_y} = \frac{z}{q_z}, \quad \frac{X-X_A}{Q_X} = \frac{Y-Y_B}{Q_Y} = \frac{Z}{Q_Z},$$

where B and A are the points of intersection of q and Q with the coordinate planes *oxy* and *OXY* respectively, q_x, q_y, q_z and Q_X, Q_Y, Q_Z are the direction cosines of line q in *oxyz* and line Q in *OXYZ* respectively.

With the assumed denotations the angle γ_{ji} between the line q_{ji} of the given j-th line set and axis Q can be determined from the expression

$$cos\gamma_{ji} = q_{jiX}Q_X + q_{jiY}Q_Y + q_{jiZ}Q_Z \ , \tag{3}$$

where q_{jiX}, q_{jiY}, q_{jiZ} denote direction cosines of q_{ji} with respect to the fixed coordinate axes and can be expressed in q_x, q_y, q_z by means of the given 3×3 rotation matrix T_{ji}:

$$\left[q_{jiX}, q_{jiY}, q_{jiZ}\right]^t = T_{ji}\left[q_x, q_y, q_z\right]^t. \tag{4}$$

Angle γ_{ji} is determined from (3) by an inverse trigonometric function of the unknown direction parameters Q_x, Q_y, Q_z that brings to a highly nonlinear expression of the angular deviation (1). Therefore, we follow the well-known approach of the approximate synthesis theory [5] and replace (1) by an algebraic deviation function:

$$\Delta_{q_{ji}} = cos\gamma_{ji} - cos\gamma_i = q_{jiX}Q_X + q_{jiY}Q_Y + q_{jiZ}Q_Z - cos\gamma_j, \tag{5}$$

where unknown Q_X, Q_Y, Q_Z are subject to the condition $Q_X^2 + Q_Y^2 + Q_Z^2 = 1$.

To estimate the "closeness" of the given line sets$\left\{q_{ji}\right\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$ to the sought-for coaxial CC congruences K_j $(j = 1,2, \dots, m)$, we use by analogy with [2] the sums of squared deviations (5) and (2) written for all the given position-sets $\left\{e_{ji}\right\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$. Accordingly, the sought-for CC congruence set$K^* = \left\{K_j^*\right\}_{j=1}^{m}$ will be determined from the necessary conditions for a minimum of the following sums:

$$S_1 = \sum_{j=1}^{m}\sum_{i=1}^{N_j} \Delta_{q_{ji}}^2, \tag{6}$$

$$S_2 = \sum_{j=1}^{m}\sum_{i=1}^{N_j} \Delta_{ji}^2. \tag{7}$$

As mentioned above, the synthesis of the CCCmechanism in Fig.1 requires to find m CC dyads with the common fixed axis Q of their pivotal cylindric joints and different (adjustable) twist angles γ_j and distances d_j between the moving axes q_j and fixed axis Q. It can be easily shown that associated with each of these dyads, there is a spherical indicatrix – a conditional RR dyad with the intersecting axes of rotation which will reproduce the same angular relationships and relative rotations as in the CC dyad under consideration [2,3]. Therefore, the formulated above problem of synthesis of C<u>C</u>C chain can be decomposed into 2 simpler subproblems: a) synthesis of its spherical indicatrix to determinate the twist angles γ_j $(j = 1,2,\dots,m)$ and directions of Q and q in systems XYZ and xyz respectively, b) synthesis of coaxial CC dyad with the known directions of axes Q and q to determine their locations in corresponding coordinate systems and distances d_j $(j = 1,2,\dots,m)$.

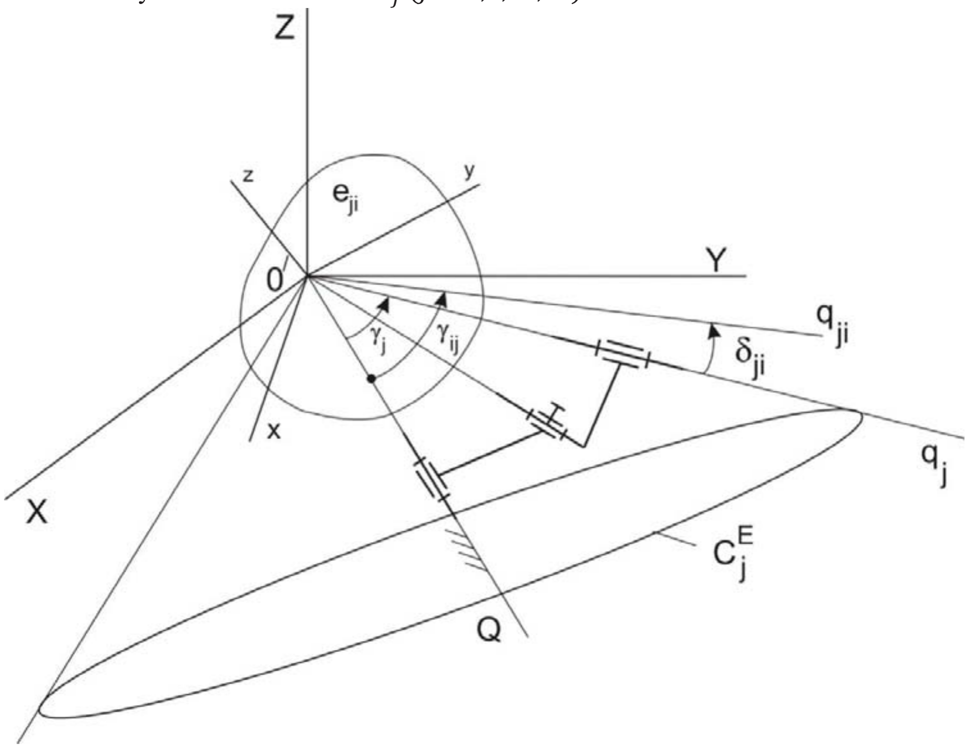

Fig.3. Spherical indicatrix of C<u>C</u>C chain for the j-th position-set

If we deal now with the problem (a) we can think of an R<u>R</u>R chain with all three revolute axes intersecting at the fixed point $0^/$ of the spherical displacements where the origin can be placed and therefore all $d_j = 0, d_{ji} = 0, X_{o_{ji}} = Y_{o_{ji}} = Z_{o_{ji}} = = 0$ (Fig.3). Since we are now concerned only with the angular part of the displacement, the m sets of given positions q_{ji} turn into m bundles of lines to be approximated by m coaxial second order circular cones C_j^E $(j = 1,2,\dots,m)$ embedded in *E*. The theory of the loci associated with the least square approximations of a spherical motion given by a single set of displacements has been developed in [5,6]. Most of the results presented in these references are valid also for *m*>1.

Defining lines$q$ and Q in the new spherical coordinate systems O$^{/}$xyz and O$^{/}$XYZ respectively by their points $l(x_l, y_l, 1)$ and $L(X_L, Y_L, 1)$, we can represent expression (5) for Δ_{ji} in the more convenient polynomial form:

$$\Delta_{q_{ji}} = X_{l_{ji}} X_L + Y_{l_{ji}} Y_L + Z_{l_{ji}} + H_j, \tag{8}$$

where $H_j = -\bar{r}_l \cdot \bar{r}_L cos\gamma_j$ is a constant depending only on the unknown design parameters, while $X_{l_{ji}}$, $Y_{l_{ji}}$ and $Z_{l_{ji}}$ are determined by the same linear transformation formula (4):

$$\left[X_{l_{ji}}, Y_{l_{ji}}, Z_{l_{ji}}\right]^t = T_{ji}[x_l, y_l, 1].$$

For each line q (point $l(x_l, y_1, 1)$) given in e, we should find the set of coaxial circular cones $C_j^E \, (j = 1,2, \dots, m)$ approximating best the corresponding sets of the prescribed line-positions$\{q_{ji}\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$. We determine this set from the necessary conditions for a minimum of (6):

$$\partial S_1/\partial X_L = 0, \quad \partial S_1/\partial Y_L = 0, \; \partial S_1/\partial H_j = 0 \; (j = 1,2, \dots, m). \tag{9}$$

Substituting (8) in (6) and denoting for brevity $X_{l_{ji}} = X_{ji}, Y_{l_{ji}} = Y_{ji}, Z_{l_{ji}} = Z_{ji}$, we can reduce conditions (9) to the following system of *(2+m)* linear equations in X_L, Y_L and H_j $(j = 1,2, \dots, m)$ presented below in a matrix form:

$$M^E p^E = F^E, \tag{10}$$

where

$$M^E = \begin{bmatrix} \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}^2 & \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji} Y_{ji} & \sum_{i=1}^{N_1} X_{1i} & \dots & \sum_{i=1}^{N_m} X_{mi} \\ \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji} Y_{ji} & \sum_{j=1}^{m}\sum_{i=1}^{N_j} Y_{ji}^2 & \sum_{i=1}^{N_1} Y_{1i} & \dots & \sum_{i=1}^{N_m} Y_{mi} \\ \sum_{i=1}^{N_1} X_{1i} & \sum_{i=1}^{N_1} Y_{1i} & N_1 & \dots & 0 \\ \sum_{i=1}^{N_2} X_{2i} & \sum_{i=1}^{N_1} Y_{1i} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{N_m} X_{mi} & \sum_{i=1}^{N_m} Y_{mi} & 0 & \dots & 0 \end{bmatrix},$$

$$p^E = [X_L, Y_L, H_1, \dots, H_m]^t,$$

$$F^E = -\left[\sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}\, Z_{ji}\,, \sum_{j=1}^{m}\sum_{i=1}^{N_j} Y_{ji}\, Z_{ji}\,, \sum_{i=1}^{N_1} Z_{1i}, \dots, \sum_{i=1}^{N_m} Z_{mi}\right]^t.$$

We present the solution of (10) in the vector form by means of Cramer's rule:

$$(X_L,\ Y_L,\ H_1, \dots, H_m) = \frac{1}{D}\left(D_x, D_y, D_{H_1}, \dots, D_{H_m}\right), \tag{11}$$

where $D_x, D_y, D_{H_1}, \dots, D_{H_m}$ are the *(2+m)*-th order determinants defined from (10) by Cramer's rule.

Equation (11) determines the sought-for set of m coaxial circular cones $\{C_j^E\}$ approximating, in the least square sense, the given m line-positionsets $\{q_{ji}\}_{i=1}^{N_j} (j = 1,2, \dots, m)$ in spherical motion of e. Solution (11) of system (10) exists and is uniquely determined unless its coefficient matrix is singular (D=0). In [6], it has been shown that $D>0$ for all points of the plane $z=1$ in e if the numbers of the given positions $N_j>4$. Knowing the coordinates X_L, Y_L, 1 of point L, we compute the direction cosines Q_X, Q_Y, Q_Z of the determined common axis L.

The cone angles γ_j of approximating coaxial cones C_j^E are determined from the expression of H_j:

$$\gamma_j = arccos\left|H_j\right| / \overline{r_l} \cdot \overline{r_L} (j = 1,2, \dots, m).$$

Now we proceed to the solution of problem (b): determination of the position of Q in *OXYZ* and distances d_j $(j = 1,2, \dots, m)$ of coaxial CC congruences K_j for the known direction (Q_X, Q_Y, Q_Z) of Q and angles γ_j $(j = 1,2, \dots, m)$. The distances d_{ji} between lines q_{ji} and axis Q can be written as:

$$d_{ji} = \frac{1}{sin\gamma_{ji}} \begin{vmatrix} X_A - X_{B_{ji}} & Y_A - Y_{B_{ji}} & -Z_{B_{ji}} \\ q_{jiX} & q_{jiY} & q_{jiZ} \\ Q_X & Q_Y & Q_Z \end{vmatrix}, \tag{12}$$

where q_{jiX}, q_{jiY}, q_{jiZ} are computed by (4), while $X_{B_{ji}}, Y_{B_{ji}}$, $Z_{B_{ji}}$ can be determined by using the linear transformation

$$\left[X_{B_{ji}}, Y_{B_{ji}}, Z_{B_{ji}}\right]^t = \left[X_{O_{ji}}, Y_{O_{ji}}, Z_{O_{ji}}\right]^t + T_{ji}[x_B, y_B, 0]^t.$$

Substituting in (2) expression (12) for d_{ji}, after some transformations, we present the distance deviation Δ_{ji} in the form of a linear function with respect to the unknown design parameters X_A, Y_A, $d_j (j = 1,2, \dots, m)$:

$$\Delta_{ji} = d_{ji} - d_j = f_{1ji}X_A + f_{2ji}Y_A - d_j + F_{ji}, \tag{13}$$

where

$$f_{1ji} = \frac{1}{sin\gamma_{ji}}\left(q_{jiY}Q_Z - q_{jiZ}Q_Y\right),\ \ f_{2ji} = \frac{1}{sin\gamma_{ji}}\left(-q_{jiX}Q_Z + q_{jiZ}Q_X\right),$$

$$F_{ji} = \frac{1}{sin\gamma_{ji}}\left[Q_X \cdot \left(\overline{q_{ji}} \times \overline{R_{B_{ji}}}\right)_X + Q_Y \cdot \left(\overline{q_{ji}} \times \overline{R_{B_{ji}}}\right)_Y + Q_Z \cdot \left(\overline{q_{ji}} \times \overline{R_{B_{ji}}}\right)_Z\right].$$

The sought-for position parameters will be determined from the necessary conditions for a minimum of (7):

$$\partial S_1 / \partial X_L = 0,\ \ \partial S_1 / \partial Y_L = 0,\ \partial S_1 / \partial H_j = 0\ (j = 1,2, \dots, m).$$

In view of (13), conditions (14) can be reduced to the following system of *(2+m)* linear equations:

$$
\begin{bmatrix}
\sum_{j=1}^{m}\sum_{i=1}^{N_j} f_{1ji}^2 & \sum_{j=1}^{m}\sum_{i=1}^{N_j} f_{1ji}f_{2ji} & -\sum_{i=1}^{N_1} f_{11i} & \dots & -\sum_{i=1}^{N_m} f_{1mi} \\
\sum_{j=1}^{m}\sum_{i=1}^{N_j} f_{1ji}f_{2ji} & \sum_{j=1}^{m}\sum_{i=1}^{N_j} f_{2ji}^2 & -\sum_{i=1}^{N_1} f_{21i} & \dots & -\sum_{i=1}^{N_m} f_{2mi} \\
-\sum_{i=1}^{N_1} f_{11i} & -\sum_{i=1}^{N_1} f_{12i} & N_1 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & 0 \\
-\sum_{i=1}^{N_m} f_{1mi} & -\sum_{i=1}^{N_m} f_{2mi} & 0 & \dots & N_m
\end{bmatrix}
\begin{bmatrix} X_A \\ Y_A \\ d_1 \\ \vdots \\ d_m \end{bmatrix}
= -
\begin{bmatrix}
\sum_{j=1}^{m}\sum_{i=1}^{N_j} f_{1ji}F_{ji} \\
\sum_{j=1}^{m}\sum_{i=1}^{N_j} f_{2ji}F_{ji} \\
\sum_{i=1}^{N_1} f_{1i} \\
\vdots \\
\sum_{i=1}^{N_1} f_{mi}
\end{bmatrix}. \quad (14)
$$

It can be easily shown that the coefficient determinant of system (14) is always positive [5] and the system has a unique solution. It follows then that for any given line q in e systems (10) and (14) determine a unique set $\{K_j\}_{j=1}^{m}$of CC congruences with the common axis Q which approximate m given sets of line-positions $\{q_{ji}\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$. Equations (10) and (14) establish correspondence between lines in *e* and the central axes of coaxial CC congruences$\{K_j^E\}_{j=1}^{m}$ in *E*.

The same arguments are also true for the inverted motion when *e* becomes the fixed body and *E* moves so as to maintain the same relative positions as in the original motion given by the initial data on the m prescribed position-sets. Here we should find such CC congruence set $\{k_j^e\}_{j=1}^{m}$ in *e* which will approximate the *m* alternating sets of the inverted positions of a line Q of *E*. First we solve the directional part of the problem (subproblem (a)) for the spherical indicatrix of C<u>C</u>C chain in the inverse motion. The direction parameters of the common axis q of $\{k_j^e\}_{j=1}^{m}$ and constants h_j $(j = 1,2, \dots, m)$ are determined from the stationary conditions of the sum (6):

$$\partial S_1/\partial x_l = 0,\ \ \partial S_1/\partial y_l = 0,\ \partial S_1/\partial h_j = 0\ (j = 1,2, \dots, m). \qquad (15)$$

Then we compute the direction parameters q_x, q_y, q_z and cone angles $\gamma_j (j=1,2,\dots,m)$ for approximating coaxial cones $C_j^E$to determine the position of q in *oxyz* and distances d_j $(j = 1,2, \dots, m)$ fork_j^e by using the stationary conditions of the sum (7):

$$\partial S_2/\partial x_B = 0,\ \ \partial S_2/\partial y_B = 0,\ \partial S_2/\partial d_j = 0\ \ (j = 1,2, \dots, m). \qquad (16)$$

Equations (15) and (16) yield two systems of linear equations similar to (10) and (14) which define a unique approximating CC congruence set $\{k_j^e\}$ for any line Q of *E* in its inverse motion.

Lines of e deviating least from the associated approximating coaxial CC congruences. In the foregoing, we have established one to one correspondence between the lines in *e* and the central axes Q of the coaxial CC congruences approximating alternating position-sets of these lines in given spatial displacements of *e*. Now we proceed to the main subject of this study: determination of those special lines in *e* which remain as close as possible to their associated approximating coaxial

CC congruence sets $\{K_j^E\}$ in all m given position sets of e. As already mentioned, the directions of lines q and Q are determined independently of their positionsin the moving and fixed coordinate systems.

Algebraic deviation $\Delta_{q_{ji}}$ given by (5) is a function of *(5+m)* design variables: X_L, Y_L, x_l, y_l, γ_j $(j = 1,2, \dots, m)$. Therefore, the direction parameters of lines q and Q for which the sum (6) attains a minimum (extremum) should satisfy the conditions.

$$\partial S_1/\partial X_L = 0, \quad \partial S_1/\partial Y_L = 0, \quad \partial S_1/\partial x_l = 0, \quad \partial S_1/\partial y_l = 0,$$
$$\partial S_1/\partial H_j = 0 \; (j = 1,2, \dots, m). \tag{17}$$

The system (17) combines stationary conditions (9) and (15) respectively for the original and inverse spherical motions of e and E. This brings to the assertion presented as the following theorem:

Theorem 1. In order for a moving line-fixed axis pair (q^*, Q^*) intersecting at point $O^/$ (Fig.3) to cause the sum (6) to be a minimum (extremum) it is necessary that:
1) Q^*be the axis of the coaxial cones C_j^E $(j = 1,2, \dots, m)$ approximating m sets (bundles) of positions $\{q_{ji}^*\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$ of line q^* in m given sets of spherical displacements of e with respect to E;
2) q^* be the axis of the coaxial cones C_j^e $(j = 1,2, \dots, m)$ approximating m sets (bundles) of inverted positions $\{Q_{ji}^*\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$ of Q^* with respect to e.

Theorem 1 is the extension of the similar theorem on the interrelation of directions of lines q and Q in spherical motion of e for a single set of given positions *(m=1)* [6] to the general case of multiple position-sets (m>1).

For the known directions of q and Q the linear (distance) deviations Δ_{ji} given by (13) and their squired sum (7) are functions of *(5+m)* variables: X_A, Y_A, x_B, y_B, d_j $(j = 1,2, \dots, m)$. Therefore, the following conditions are necessary for a minimum (extremum) of (7):

$$\partial S_2/\partial X_A = 0, \quad \partial S_2/\partial Y_A = 0, \quad \partial S_2/\partial x_B = 0, \quad \partial S_2/\partial y_B = 0,$$
$$\partial S_2/\partial d_j = 0 \; (j = 1,2, \dots, m). \tag{18}$$

System (18) unites the stationary conditions (15) and (16) for S_2 written for the original and inverse relative motions of e and E. The joint analysis of equations (17) and (18) together with the assertion of Theorem 1 permitsto present the necessary conditions for a pair of lines q and Q to minimize the sum (17) and (18) in the form of the following theorem.

Theorem 2. In order for a moving line-fixed axis pair (q^*, Q^*) to cause the sum (6) and (7) to be a minimum (extremum) it is necessary that:
1) Q^* be the central axis of m coaxial CC congruenses $\{K_j^E\}_{j=1}^m$ approximating m position-sets $\{q_{ji}^*\}_{i=1}^{N_j}$ $(j = 1,2, \dots, m)$ in the given m sets of spatial displacements of e relative to E;

2) q^* be the central axis of m coaxial CC congruences $\{k_j^e\}_{j=1}^{m}$ approximating the m sets of the inverted positions $\{Q_{ji}^*\}_{i=1}^{N_j}$ $(j=1,2,\ldots,m)$ of Q^* in the inverse motion of E relative to e.

Equation (17) for determining the directions of q and Q can be reduced to a set of two *9-th* order algebraic equations in x_B and y_B. For the case when we are given a single set of positions $\{e_i\}_{i=1}^{N}$ $(m=1)$ it has been shown that this set may have at most 33 real solutions and therefore it can define up to 33 coupled directions for q and Q corresponding to the extremums of the sum (6). A simple analysis has shown that this property does not depend on the number of the given position-sets. With the known directions of q and Q, system (18) can be transformed into a system of *(4+m)* linear equations which uniquely determines the positions of sought-for lines q and Q in systems *xyz* and *XYZ* respectively, as well as the adjustable values d_j $(j=1,2,\ldots,m)$ of distance d between them. The computational part of this study and the numerical results of synthesis of the adjustable robotic mechanisms made of C<u>C</u>C modules considered above will be presented in a companion paper of the author.

Conclusion. We have presented a study of special lines in a moving body which in the m alternating sets of its ordered positions deviate least, in a least-square sense, from the coaxial line congruences generated by the moving axes of C<u>C</u>C chain with the middle joints locked in all operation modes of the mechanism serving for adjusting the distance and twist angle between the moving axis q and fixed axis Q in accordance with the changing kinematic tasks performed by the mechanism. The results presented here can be regarded as an extension of the theory developed in [2] for a single set of given positions to the general case of multiple sets. The results of this study can be readily applied for building reconfigurable parallel mechanisms made of C<u>C</u>C modules.

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ՔԱՌԱԿՈՒՍԱՅԻՆ ՄՈՏԱՐԿՈՒՄՆԵՐ ԳԾԵՐԻ ՎԵՐՋԱՎՈՐ ԲԱԶՄՈՒԹՅՈՒՆՆԵՐՈՒՄ՝ ԿԱՐԳԱՎՈՐՎՈՂ ՄԱՆԻՊՈՒԼՅԱՑԻՈՆ ՄԵԽԱՆԻԶՄՆԵՐԻ ՍԻՆԹԵԶՈՒՄ ԿԻՐԱՌՄԱՆ ՀԱՄԱՐ

Յու.Լ. Սարգսյան

Դիտարկվում է տեղաշարժվող մարմնի այնպիսի հատուկ գծերի որոշման խնդիրը, որոնք նրա նախանշված դիրքերի տրված հաջորդականություններում ամենաքիչն են շեղվում համառանցք ԳԳ (գլանային-գլանային) դիադների շարժուն առանցքների ձևավորած համառանցք գծային կոնգրուենցիաներից՝ նվազագույն քառակուսիների իմաստով: Դիտարկվող մոտարկումները նվազարկում են որոնելի գծերի մոտարկվող կոնգրուենցիաներից անկյունային և գծային շեղումների քառակուսիների երկու գումարները դիրքերի տրված բոլոր բազմությունների դեպքում: Ձևակերպվել է երկու թեորեմ, որոնք երկրաչափորեն նկարագրում են սֆերիկ և տարածական շարժումներ կատարող մարմնին պատկանող գծի դիրքերի տրված *m* հաջորդականությունների համապատասխանաբար համառանցք շրջանային կոներով և համառանցք գծային կոնգրուենցիաներով լավագույն քառակուսային մոտարկման անհրաժեշտ պայմանները: Հետազոտվել են դիտարկվող հատուկ գծերի երկրաչափական տեղերը, առաջարկվել են դրանց որոնման սկզբունքներ և ալգորիթմ: Հոդվածում զարգացվող տեսությունը կարող է անմիջականորեն կիրառվել կարգավորվող զուգահեռ մանիպուլյացիոն մեխանիզմների սինթեզում, որոնց հիմքում ընկած են ԳԳԳ (գլանային-գլանային-գլանային) տիպի կինեմատիկական շղթաներ՝ ծայրային գլանային զույգերի շարժուն և անշարժ առանցքների միջև հեռավորությունը և անկյունն ըստ կինեմատիկական առաջադրանքի փոփոխման ֆունկցիա կատարող միջին գլանային զույգով: Դիտարկվող ԳԳԳ շղթայի սինթեզի խնդիրը տրոհվում է երկու ավելի պարզ ենթախնդիրների. ա) կարգավորող միջին պտտական զույգով սֆերիկ ինդիկատրիքսի (պտտական-պտտական-պտտական (ՊՊՊ)) սինթեզ՝ պտտման շարժուն և անշարժ առանցքների ուղղությունների և կարգավորվող միջառանցքային անկյան *m* արժեքների որոշմամբ, բ) պտտման առանցքների հայտնի ուղղություններով ԳԳԳ շղթայի սինթեզ՝ համապատասխան կոորդինատային համակարգերում այդ առանցքների դիրքերի և դրանց միջև կարգավորվող հեռավորության *m* արժեքների որոշմամբ:

Առանցքային բառեր. ուղիղների համառանցք կոնգրուենցիաներ, քառակուսային մոտարկումներ, անկյունային շեղում, ԳԳ դիադ, մանիպուլյացիոն մեխանիզմ:

КВАДРАТИЧЕСКИЕ ПРИБЛИЖЕНИЯ НА КОНЕЧНЫХ МНОЖЕСТВАХ ЛИНИЙ ПРИМЕНИТЕЛЬНО К СИНТЕЗУ РЕГУЛИРУЕМЫХ МАНИПУЛЯЦИОННЫХ МЕХАНИЗМОВ

Ю.Л. Саркисян

Рассматривается задача определения специальных линий движущегося тела, которые наименее отклоняются в смысле наименьших квадратов от соосных конгруэнций прямых, генерируемых подвижными осями ЦЦ диад с общей неподвижной осью, в заданных наборах положений тела. Искомое приближение минимизирует суммы квадратов угловых и линейных отклонений указанных прямых от аппроксимирующих линейных конгруэнций, ассоциированных с синтезируемыми ЦЦ диадами. Сформулированы две теоремы, геометрически описывающие необходимые условия наилучшего квадратического приближения заданных *m* последовательностей положений линии тела, совершающего сферические и пространственные движения посредством собственно соосных круговых конусов и соосных линейных конгруэнций соответственно. Исследованы геометрические места указанных специальных линий, предложены теоретические принципы и алгоритм их определения. Развиваемая здесь теория может быть непосредственно применена в синтезе регулируемых параллельных манипуляционных механизмов, построенных на базе ЦЦ цепей с блокируемыми срединными парами, служащими для переналадки угла и расстояния между осями опорной и подвижной цилиндрических пар в соответствии с изменениями кинематического задания. Задача синтеза рассматриваемой ЦЦЦ цепи разбивается на две более простые подзадачи: а) синтез ее сферической индикатрисы ВВВ с блокируемой (регулирующей) срединной вращательной парой с определением направлений осей подвижной и неподвижной вращательных пар и *m* значений регулируемого угла между ними; б) синтез ЦЦЦ цепи при известных направлениях указанных осей вращения с определением их положений в соответствующих системах координат и m значений регулируемого расстояния между ними.

Ключевые слова: соосные конгруэнции прямых, квадратическое приближение, угловое отклонение, ЦЦ диада, манипуляционный механизм.